A Ratio Transformer Bridge for Standardization of Inductors and Capacitors

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A Ratio Transformer Bridge for Standardization of Inductors and Capacitors*

D. L. HILLHOUSE† AND H. W. KLINE†

INTRODUCTION

IN the measurement of inductance and capacitance at audio frequencies, accuracies better than 0.1 per cent are how being demanded. As an Electrical and Physical Standards operation serving the entire General Electric Company, it has been necessary for us to improve standards and techniques in order to keep ahead of these demands. Present standards consist of air core inductors at decade levels from 100 μh to 10 henries, and air and silvered-mica capacitors at decade levels from 100 pf to 1 μf. Each one of this group of standards has been certified periodically by the National Bureau of Standards. The best certifications are 0.03 per cent for inductors and 0.01 per cent for certain capacitors. With the available measuring equipment, the full accuracy of these standards has not been utilized except in a direct 1:1 comparison.

Stimulated by two papers[1,2] presented at the last Boulder Conference, we decided upon construction of a ratio transformer bridge to meet this increased accuracy requirement. One objective was an accuracy of at least 0.01 per cent at 1 kc over as much of the above mentioned range as possible. Although certification accuracies better than 0.01 per cent were not then available from the National Bureau of Standards, it was anticipated that higher accuracy measurements and certifications might soon be possible.[1,2] Another objective was the reduction of direct reliance on NBS for certifications. A third objective was the extension of capacitance standards to 0.01 pf.[3]

DESIGN AND CONSTRUCTION

The bridge is composed of a tunable indicating amplifier null detector, a 500-ohm, 3-watt audio generator, twin cubicles into which the standards are inserted for comparison, a decade resistance box and a ratio transformer having a ratio accuracy approaching 1 PPM. The design features the use of commercially available equipment. All components except the detector are combined into one integral unit.

The components are connected as shown in the wiring diagram, Fig. 1. RG58A/U coaxial cable and BNC connectors are used for wiring. Two bridge supply transformers are used to match the output impedance of the generator to various bridge loads.

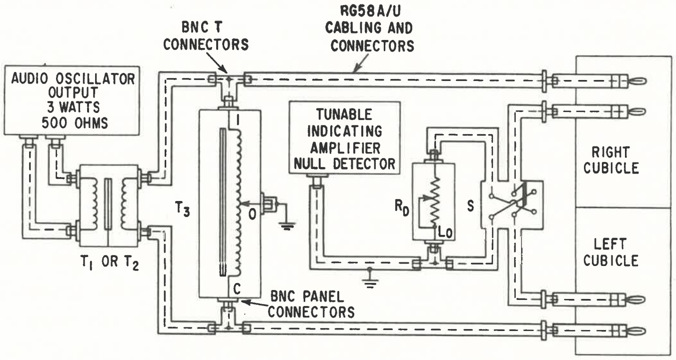
BNC T CONNECTORS
RG58A/U CABLING AND CONNECTORS
AUDIO OSCILLATOR OUTPUT 3 WATTS 500 OHMS
TUNABLE INDICATING AMPLIFIER NULL DETECTOR
RIGHT CUBICLE
LEFT CUBICLE
I
O
C
T3
T1 OR T2
RD
S
LO
BNC PANEL CONNECTORS

Fig. 1—Ratio transformer bridge wiring diagram. T_1=transformer 1:4 or 4:1; T_2=transformer 500 ohms to variable 0.06- to 8-ohm secondary; T_3=ratio transformer having ratio accuracy to 1 part in 10^6; R_D=0.1 to 111,111.2-ohm decade, resistor (6 dials); S=DPDT switch to connect R_D to either right or left cubicle.

One transformer may be connected with a ratio of 1:4 or 4:1. The 500-ohm generator output may be thus transformed to either 30 or 8000 ohms. This transformer is of the shielded, balanced type designed particularly for operation at high-impedance levels. The remaining transformer is used to step down the generator impedance to 11 levels from 8.0 to 0.06 ohms, adjustable in steps by a switch mounted in the same box with the transformer. It need not be specially shielded since stray effects are negligible at such low-impedance levels. An ordinary line-to-voice coil transformer was used. The principal purpose of approximate impedance matching is to realize good bridge sensitivity. A 0.6-ohm load (100 μh at 1 kc) would be a short circuit on a 500-ohm output and there would be very little voltage left across the load. This combination of impedance levels is adequate for good bridge sensitivity for all combinations of test load impedance down to a few microhenries.

Phase balance in nulling is obtained by means of the resistor R_D, Fig. 1. The low-resistance decade is at the end of the decade box connected to the null detector in order to minimize ground capacitance effects. The switch S facilitates connecting the decade resistor to either the right or left cubicle, depending on which reactance has the higher Q. This increases the flexibility of the bridge in that a certified standard can be connected in either cubicle and a measurement cross-checked. The test leads have adapters which may be installed for three-terminal measurements.

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[1] A. M. Thompson, "The precise measurement of small capacitances," IRE TRANS. ON INSTRUMENTATION, vol. I-7, pp. 245–253; December, 1958.

[2] M. C. McGregor, J. F. Hersh, R. D. Cutkosky, F. K. Harris, and F. R. Kotter, "New apparatus at the National Bureau of Standards for absolute capacitance measurements," IRE TRANS. ON INSTRUMENTATION, vol. I-7, pp. 253–261; December, 1958.

[3] S. C. Richardson, "Reference level standards in a large electrical manufacturing company," *Proc. ISA, 14th Annual Instrument-Automation Conf. and Exhibit,* vol. 14, pt. 1 (to be published).

OPERATING CHARACTERISTICS

Important considerations in any bridge are ratio accuracy, ease of operation, flexibility, and readability. The elimination of auxiliary balances contributes greatly to the ease of operation of this bridge. Auxiliary balance of the usual ac bridge is made necessary by the ground impedances indicated in Fig. 2(a) as Z_{g1} and Z_{g2}. These impedances are in shunt around arms Z_1 and Z_2 and will cause error in the bridge ratio unless $Z_{g1}/Z_{g2}=Z_1/Z_2$, or Z_1 and $Z_2 \ll Z_{g1}$ and Z_{g2}. The usual method of avoiding this error is to remove the bridge entirely from ground, as shown in Fig. 2(b), and balance out the ground impedances with an auxiliary grounded network, Z_5 and Z_6. This requires extra circuit components and two balancing operations, each of which may itself be a dual balance.

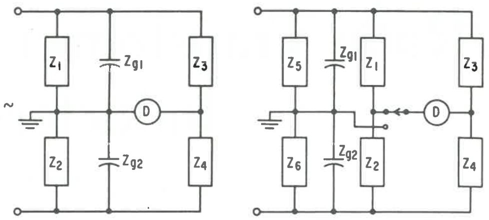
The ratio transformer has extremely high input impedance and extremely low output impedance. The low output impedance effectively shorts out the ground capacitances so that no auxiliary balance is necessary. At the same time, the transformer does not load the source. This can be proved by reference to Fig. 3. Fig. 3(a) shows the basic circuit of the auto-transformer with a capacitance load C_g, which represents the unbalanced ground capacitance in the bridge application. The circuit fits the general case of two mutually coupled self inductances, connected aiding. The equivalent circuit is shown in Fig. 3(b). The quantity of interest is E_2. It can be shown (see Appendix) that

$$E_2 = \frac{E(L_2 + M)}{(L_1 + L_2 + 2M) - \omega^2 C_g (L_1 L_2 - M^2)} \cdot \tag{1}$$

In (1), the term containing C_g disappears if $(L_1L_2 - M^2) = 0$. Applying this condition, remembering that $M = k\sqrt{L_1L_2}$, where k is the coefficient of coupling,

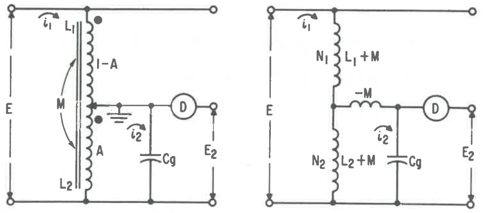
$$\begin{aligned} L_1L_2 &= k^2 L_1 L_2 \\ k &= 1. \end{aligned} \tag{2}$$

Therefore, for complete elimination of the effect of unbalanced ground capacitance on E_2, the coefficient of coupling must equal unity, *i.e.*, the leakage reactance must be zero. This condition is so closely approached in present ratio transformers that C_g has no effect unless it becomes very large. At the same time, the open-circuit input impedance is $j\omega(L_1+L_2+2M)$, a very large quantity. The size of C_g which will cause ratio error is discussed in the Section on Sources of Error.



(a) (b)

Fig. 2—General ac bridge. (a) With uncompensated ground impedances. (b) With auxiliary balancing network.



(a) (b)

Fig. 3—(a) Basic ratio transformer circuit. (b) Equivalent circuit.

Another desirable characteristic for ease of operation is that the two components being balanced are essentially in quadrature. The transformer bridge satisfies this requirement very well for all Q's greater than one. This is demonstrated by the vector diagram in Fig. 5(a) which is drawn for a Q of unity. Point D is the transformer tap on vector E. Vector DD' represents the bridge unbalance voltage, e_d. As the transformer ratio is adjusted, point D slides along vector E. As the resistive component is adjusted, point D' slides essentially along vector IR_X. Thus, e_d reduces in magnitude very rapidly. At low Q's, the approach to null becomes what is known as a "sliding balance." Nulling can then become quite tedious, and even subject to error, unless the so-called "overshoot"[4,5] principle is used. This technique consists of going deliberately beyond the minimum on each balancing operation. It has been used to measure the lead inductances of the bridge itself (of the order of 1 μh with a Q of about 0.06) to five-place precision against a 100-μh standard at 1 kc.

Another outstanding characteristic of the ratio transformer bridge is its accurate and continuously variable adjustment. The setting can be read to less than 1 PPM and is accurate to almost 1 PPM for ratios of 1:10 to 10:1. Thus, with one standard it is possible to cover a spread of 100:1. Such accuracies are also made possible by the extremely low leakage reactance. With C_g eliminated, (1) reduces to

$$E_2 = \frac{E(L_2 + M)}{L_1 + L_2 + 2M} \cdot \tag{3}$$

It can be shown that if the coefficient of coupling approaches unity, then (see Appendix)

$$A = \frac{E_2}{E} = \frac{N_2}{N_1 + N_2} \cdot \tag{4}$$

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[4] F. K. Harris, "Electrical Measurements," John Wiley and Sons, Inc., New York, N. Y., p. 698; 1952.

[5] G. B. Hoadley, "The science of balancing an impedance bridge," *J. Franklin Inst.*, vol. 228, no. 6, pp. 733–754; December, 1939.

This is the voltage transformation ratio of the perfect autotransformer.

Another desirable characteristic is direct readout. This bridge does not read out directly, but in standardizing, where the impedances seldom stray very far from nominal values, this is a minor inconvenience. Usually the exact value of a nominal ratio comparison, such as 1:1, 2:1, 5:1, or 10:1, can be obtained to 1 PPM or better by simple additions or subtractions and a slide rule calculation. (See the Appendix for an example.) Residual corrections, such as lead inductances, can be tabulated. Another possibility is the tabulation of all ratios within a few tenths per cent of the exact nominal ratio. Even without these shortcuts, the only precise operation required is a single calculation to obtain the true ratio.

OPERATING EQUATIONS

The equations for determining the values of inductance and effective series resistance are derived from Fig. 4(a) and shown in Table I. In this instance, the standard inductor L_S is in the right cubicle. Altogether, there are four possible equations, depending on whether the standard inductor and the decade resistor are on either the right or left side.

Values of wiring, lead and decade resistance box inductances are also tabulated, primarily to indicate their order of magnitude for this particular bridge. Corrections for these inductances become necessary below 20 mh in order to reach accuracies better than 0.01 per cent in a 10:1 measurement.

TABLE I

BRIDGE OPERATING EQUATIONS FOR INDUCTANCE

<table>
<tr><th colspan="4">Inductance</th><th colspan="3">Effective Resistance</th></tr>
<tr><td colspan="4">$\frac{1-A}{A}=\frac{\text{Total } L \text{ in right arm}}{\text{Total } L \text{ in left arm}}$</td><td colspan="3">$\frac{1-A}{A}=\frac{\text{Total } R \text{ in right arm}}{\text{Total } R \text{ in left arm}}$</td></tr>
<tr><td colspan="7">Solutions for the Connection in Fig. 4(a)</td></tr>
<tr><td colspan="4">$\frac{1-A}{A}=\frac{L_S+L_r+L_D}{L_X+L_l}$
$L_X=\frac{A(L_S+L_r+L_D)}{1-A}-L_l \quad (5)$</td><td colspan="3">$\frac{1-A}{A}=\frac{R_S+R_r+R_D}{R_X+R_l}$
$R_X=\frac{A(R_S+R_r+R_D)}{1-A}-R_l \quad (6)$</td></tr>
<tr><td colspan="7">Wiring and Lead Values (Constant)</td></tr>
<tr><td colspan="4"><i>Microhenries</i></td><td colspan="3"><i>Ohms</i></td></tr>
<tr><td></td><td><i>Sw to r</i></td><td><i>Sw to l</i></td><td></td><td></td><td><i>Sw to r</i></td><td><i>Sw to l</i></td></tr>
<tr><td>L_l</td><td>1.23</td><td>1.73</td><td></td><td>R_l</td><td>0.067</td><td>0.169</td></tr>
<tr><td>L_r</td><td>1.71</td><td>1.18</td><td></td><td>R_r</td><td>0.169</td><td>0.067</td></tr>
<tr><td colspan="7">Decade Resistance Box Values (Variable)</td></tr>
<tr><td colspan="4"><i>Microhenry/Step</i></td><td colspan="3" rowspan="4">R_D = Dial Readings</td></tr>
<tr><td></td><td><i>1.1 slidewire</i></td><td><i>10 dial</i></td><td><i>100 dial</i></td></tr>
<tr><td>$\Delta L_D=$</td><td>0</td><td>0.056</td><td>0.11</td></tr>
<tr><td colspan="4">$L_D=\Delta L_D\times$ number of steps cut in.</td></tr>
</table>

The operating equations for capacitance, Table II, are also obtained from Fig. 4(a) by the same line of reasoning as shown in Table I. Corrections for lead inductance must be applied above 1.0 μf at 1 kc to reach accuracies better than 0.01 per cent. The same effective resistance equations apply as for inductances. However, for capacitances below 1000 pf, the series balancing circuit is not usually suitable for obtaining accurate results for capacitor resistance unless ground capacitance corrections are applied. This is discussed more fully in the next Section.

TABLE II

BRIDGE OPERATING EQUATIONS FOR CAPACITANCE

<table>
<tr><td>$\frac{1-A}{A}=\frac{\text{Resultant } C \text{ in left arm}}{\text{Resultant } C \text{ in right arm}}$</td><td></td></tr>
<tr><td>Solution for the connection in Fig. 4(a)</td><td></td></tr>
<tr><td>$C_X=\frac{C_S(1-A)}{A}+\Delta C_X$</td><td>(7)</td></tr>
<tr><td>Solution for ΔC_X for the connection in Fig. 4(a)</td><td></td></tr>
<tr><td>$\Delta C_X=\omega^2 C_X[C_S(L_r+L_D)-C_X L_l]$</td><td>(8)</td></tr>
</table>

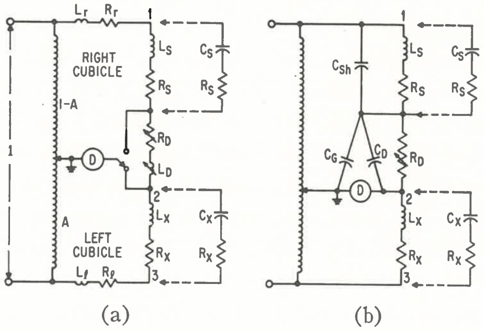

Fig. 4—(a) Circuit for derivation of bridge operating equations. (b) Circuit for derivation of capacitance errors.

SOURCES OF ERROR

A number of possible sources of error have been studied. These include 1) ground capacitance, 2) shunt capacitance, 3) series lead inductance, 4) location of the series balancing resistance, 5) lead resistance, 6) insufficient resolution of the series balancing resistance, and 7) inherent transformer ratio errors. These will now be discussed in turn.

The general effect of ground capacitance on the transformer ratio has already been discussed in some detail in connection with the characteristics of the bridge. The magnitude of this error at 1 kc was checked by deliberately loading capacitance from one side of the bridge to ground. The results for the most commonly used ratios are shown in Table III.

Maximum normal capacitance unbalance in the bridge is considerably less than 0.001 μf. Therefore, ratio error caused by ground capacitance is negligible at 1 kc.

TABLE III

Ratio Error Produced by Unbalanced Ground Capacitance

Nominal Ratio	C_g—μf	Per Cent Error
10:1	0.1	0.037
1:10	0.1	0.003
1:1	0.1	0.008

Ground capacitance associated with the series balancing resistor can cause appreciable error at high impedances. This capacitance is shown as C_G in Fig. 4(b). Its effect for inductance measurements can be evaluated by converting the T circuit formed by $(L_S + R_S)$, R_D and C_G into its equivalent Pi, ignoring C_{Sh} and C_D (see Appendix). The equivalent impedance between 1 and 2 is then compared with the impedance which would be present if C_G were absent. The difference is the error contributed by C_G. The inductance error is $(R_D R_S C_G)$. The resistance error is $(-\omega^2 R_D L_S C_G)$. To illustrate numerically the magnitudes of these errors, consider a typical inductance standard in which L_S equals 10 henries, and R_S equals 10,000 ohms. Assume that f equals 1 kc, R_D equals 1000 ohms and C_G equals 100 pf, which is its approximate value in this bridge. This assumes also a 10 per cent difference in the Q's of the two coils being compared, with the standard having the higher Q. Substituting these values into the above expressions for error yields 1 mh, or 0.01 per cent inductance error, and (-39) ohms or $(-0.39$ per cent) resistance error. Since both errors depend on R_D, they disappear if the Q's of the inductances being compared are very similar. Since R_D and R_S both tend to be proportional to L_S, the inductance error decreases roughly as L_S^2 and is usually negligible below 10 henries. Applying the same treatment to capacitance, using the series equivalent circuit, the capacitance error is $(\omega^2 R_D R_S C_G C_S^2)$. The resistance error is $(R_D C_G / C_S)$. For $C_G = 100$ pf, and for typical high-quality capacitors, the capacitance error is less than 1 ppm over the range from 1 μf to 0.01 pf. The resistance error is negligible for large capacitances, but is as large as R_D itself when C_S equals C_G. If C_S is one-tenth of C_G, the error is 10 R_D, and determining the effective series resistance becomes dependent almost entirely on the accuracy with which C_G is known. This can be turned from a disadvantage to an asset by deliberately adding a large amount of known ground capacitance. The residual C_G of the bridge is thus swamped out, making its accurate determination unnecessary. Then the conductance of small capacitors can be measured, with the additional advantage that large values of R_D are not needed, because of the multiplying effect of the ratio C_G/C_S. Since the measured capacitance is negligibly affected by C_G, the modified circuit will still give a correct answer for C_S.

Shunt capacitance around the impedance being measured adds directly to capacitance, and according to the relation $(\omega^2 L_S^2 C_{Sh})$ to inductance. It is solely a two-terminal problem, and is extremely difficult to evaluate analytically.[6] In general, no attempt is made to correct for it. It determines the boundary region for the changeover from two-terminal to three-terminal measurements. For the precision of which this bridge is capable, this boundary falls approximately at 1000 pf and 10 henries. At 1 kc, 0.1 pf in shunt affects the value of a 10-henry inductor by approximately 0.004 per cent. At 1000 pf, its effect is 0.01 per cent. In this bridge, the shunt capacitance which is not inherently part of the effective reactance being measured can be kept below 0.1 pf and held constant to better than 0.01 pf. The effects of orientation, of leads, and of removing the device altogether from the measuring compartment have been investigated. If the terminals are kept at least three inches away from the grounded compartment walls, the capacitance change caused by orientation is less than 0.005 pf. The effect increases in an exponential manner as a grounded plane is approached, so that bringing the terminals within an inch of the compartment wall can reduce the effective capacitance as much as 0.1 pf. Inserting the special test leads into the capacitor terminals reduces the effective capacitance by about 0.03 pf. Removing the device entirely from the measuring compartment produces a capacitance increase of 0.04 to 0.05 pf. From this, it appears that two-terminal capacitance values obtained on this bridge are within 0.07 to 0.08 pf of the free capacitance. The above results also emphasize the great importance of termination, orientation, and spacing in attempting two-terminal measurements where 0.1 pf is significant. Shunt capacitance puts an accuracy limitation of approximately 0.01 to 0.05 pf on such measurements (the exact estimate seems to depend on the experience and conservatism of the experimenter). The only exception is a 1:1 comparison where these effects cancel because the two measuring compartments and the leads are identical.

Shunt capacitance around R_D produces an inductance error which is approximately equal to $(-R_D^2 C_D)$; and a resistance error equal to $(\omega^2 R_D^2 R_S C_G C_D)$. With the same numerical values as used earlier in the discussion of C_G, and with an assumed C_D of 100 pf, the resultant inductance error is $(-0.001$ per cent). This is opposite in sign to the error produced by ground capacitance. Therefore, it is not only quite small, but tends to be cancelled by ground capacitance error. With the same parameters, C_D produces an error of less than 1 PPM in the equivalent coil resistance.

Series lead inductances can be significant sources of error when measuring low inductances or high capacitances. They add directly to inductance, and according to the relationship $(\omega^2 L_r C_s^2)$ to capacitance. Lead inductance unbalance of 1 μh in a 1:1 measurement at 1 kc creates an error of 0.01 per cent at 10 mh, or at 2.5 μf. The actual lead inductances for this bridge are of the order of 1 to 2 μh, and therefore must be measured very carefully and corrections applied in order to work to high accuracy at the 100-μh level. The uncertainty of lead inductance measurements is believed to be about 0.02 μh. At a bridge ratio of 10:1, virtually all of the lead inductance on one side of the bridge becomes a source of error. Eq. (5) shows the application of these lead corrections to inductance measurement, taking account of the bridge ratio. The equivalent capacitance correction ΔC_X, as given in (8), is derived in the Appendix by reference to Fig. 4(a).

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[6] J. F. Hersh, "A close look at connection errors in capacitance measurements," *General Radio Experimenter*, vol. 33, pp. 3–12; July, 1959.

In terms of circuit effects, the series balancing resistor can equally well be located either on the high or low side of the impedance being measured. When the resulting T circuit is transposed into its equivalent Pi, the impedance between points 1 and 2 is identical to that obtained from converting the circuit of Fig. 4(b). This has been confirmed by measurements with the series resistor in both positions.

Lead resistances do not ordinarily cause any error in the capacitance or inductance measurements themselves. They become important only in the determination of the effective resistances of low inductance coils. A typical 100-μh standard will have a resistance which is of the same order of magnitude as the residual lead resistances.

Insufficient resolution of the series balancing resistance can produce error by causing the detector to bypass the true null. This has the same effect as the detector dead band or sensitivity limit. The ratio error caused by lack of resistance resolution can be shown from Fig. 5(b) (see Appendix) to be approximately

$$\Delta A = \frac{(\Delta R_X)}{2} A \frac{R_X}{|Z_X{}^2|} \tag{9}$$

where (ΔR_X) is the resolution limit of the series resistance. Resolution begins to be of concern in measuring inductances below 1 mh. For a typical 100-μh standard, in which the coil resistance is 0.2 ohm, measured against a 1-mh standard, a resolution limit of 0.007 ohm causes a ratio error of 0.001 per cent. The resolution limit for this bridge is approximately 0.005 ohm.

The ratio transformer itself has some inherent error. The manufacturer claims a terminal linearity of 1 PPM.[7] The accuracy of this particular transformer has not been determined. Symmetry of error has been proven by measuring impedances with ratios as great as 10:1 with the transformer first direct and then reversed. If the ratio transformer readings, A_1 and A_2, for two such measurements are added together, they should close to unity. The difference of (A_1+A_2) from unity is equal to the difference of the ratio errors at the two symmetrical settings. In almost all cases, this difference has been less than 1 PPM, and its maximum value was 2 PPM. Although theoretically these errors could still have any size whatsoever, it is most unlikely that they would equal each other so closely if they were much larger than a few PPM. This implies extremely high ratio accuracy, although it actually does not prove it.

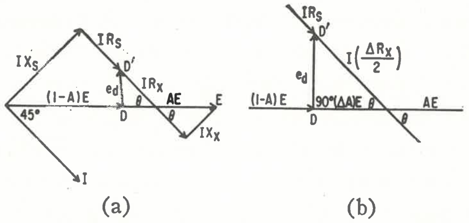

Fig. 5—(a) Vector diagram of ratio transformer bridge. (b) Magnified diagram showing residual e_d caused by lack of resistance resolution.

The combined effects of all the above errors, except series lead inductance and shunt capacitance, are estimated to be of the order of 0.001 per cent for ratios up to 10:1. Only lead inductance corrections are significant enough to be incorporated into the operating equations of the bridge. The others appear only under special conditions.

TEST RESULTS

Since there were no available standards capable of evaluating the accuracy of this bridge, the best check that could be obtained was an intercomparison to determine how much error might be introduced by the bridge in a series of 10:1 transfers running over the range of each reactance group. The results of such an intercomparison are shown in Table IV. The significant data are displayed in the Δ per cent column. The first group shows the agreement with certified values for inductors when the other standards are referred arbitrarily to the 100-mh standard. The total spread over the en-

TABLE IV

COMPARISON OF STANDARDS

Certified Values	Certified Accuracy Per Cent	Measured Values	Δ Per Cent
99.92 μh	0.1	99.93 μh	+0.01
0.9997 mh	0.03	0.99978 mh	+0.008
9.996 mh	0.03	9.9961 mh	+0.001
19.997 mh	0.03	19.9976 mh	+0.006
99.99 mh	0.03	Reference	0
1.0024 henry	0.05	1.00261 henry	+0.021
10.28 henries	0.2	10.2811 henries	+0.011
0.0100 pf	0.1	0.009989*	−0.11
0.09993 pf	0.1	0.099837*	−0.093
0.9999 pf	0.1	0.99986*	−0.004
9.9993 pf	0.1	9.9990*	−0.003
10.000 pf	0.1	10.0004*	+0.004
99.97 pf	0.1	99.986 pf	+0.016
999.8 pf	0.05	999.83 pf	+0.003
0.010012 μf	0.03	0.0100124 μf	+0.004
0.099989 μf	0.01	Reference	0
1.0010 μf	0.03	1.00106 μf	+0.006
9.992 μf†	0.05	9.989 μf	−0.03

* Preliminary measurements on new 3-terminal capacitors certified by manufacturer to tolerances indicated.

† A polystyrene decade capacitor tested and reported but not certified by NBS in May, 1959.

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[7] N. E. Morrison, "Calibration of a Decade Transformer Having One PPM Linearity," presented at the AIEE Summer and Pacific General Meeting, Seattle, Wash., Paper No. 59-997; June 24, 1959.

tire range from 100 μh to 10 henries is 0.021 per cent. This is well within certified values. All measurements except that at 100 μh are repeatable to a few thousandths of a per cent. Spreads of as much as 0.02 per cent were encountered at the 100-μh level. This is largely attributable to uncertainty in measuring the residual lead and wiring inductances. Ambient temperature was 23°C±0.5°C. A spread of 0.003 per cent could possibly be attributed to this.

The second group shows the comparison of other capacitance standards with a 0.1-μf capacitor certified by NBS to ±0.01 per cent. In decreasing by seven decades to 0.01 pf, the accumulated error based on certified values was only 0.1 per cent. Individual results were repeatable to within a few thousandths per cent. Preliminary results on the new capacitors are well within the manufacturer's tolerance down to 1 pf, and just at the limit below that. The 10-μf polystyrene capacitor was not certified by NBS due to lack of history. These capacitors sometimes drift, especially when new. Therefore, the −0.03 per cent deviation exhibited was probably due to drift. Within the range of NBS certified standards, the largest deviation is less than 0.02 per cent. Most of this occurs at 100 pf, and is due to termination uncertainties.

All of the above standards, except capacitors below 100 pf, were measured as two-terminal units. The listed NBS values for the 1000- and 100-pf capacitors are the NBS estimates of their free capacitances. As was discussed in the Section on Sources of Error, we feel that our termination is within a few hundredths of a picofarad of the free capacitance. Therefore, the comparison was made on this basis. The values obtained are actually higher than the NBS estimate of the free capacitances, but the difference is well within the certified tolerance for the capacitors.

The most significant point made by these data is that it is possible with this bridge to standardize over a wide range to higher accuracy than the NBS certifications. This is due to the higher accuracy certification of a few standards plus the transfer accuracy of the bridge.

Table V presents the results of inductor resistance measurements. These data are obtained simultaneously with the inductance values. They are compared with resonance bridge measurements whose estimated accuracy is shown in the third column. Taking the 10-henry value as an arbitrary reference, the accumulated deviation over six decades is in substantial agreement with the estimated resonance bridge accuracy.

TABLE V

Effective Resistance Measurements

Nominal Inductance	Resonance Bridge Values Ohms	Estimated Accuracy Per Cent	Measured Values Ohms	Δ Per Cent
10 henries	8268	0.1	Reference	0
1 henry	610.0	0.1	611.2	+0.19
100 mh	90.73	0.1	91.20	+0.52
20 mh	19.16	0.5	19.25	+0.43
10 mh	8.53	0.5	8.57	+0.40
1 mh	0.931	1	0.926	−0.56
100 μh	0.173	2	0.168	−2.5

Future Goals

One goal of this work is to reduce direct reliance on the National Bureau of Standards for AF reactance standards to a single reference group of capacitors. Fig. 6 shows tentative plans for accomplishing this. Reference standards will consist of a group of at least three capacitors, probably 0.1 μf. The 0.1-μf value is chosen because at present the best certifications from the National Bureau of Standards are obtained at this level. The reference standards could equally well be a group of air capacitors at a lower level. It is probable that the most stable capacitors will ultimately be of the latter type.

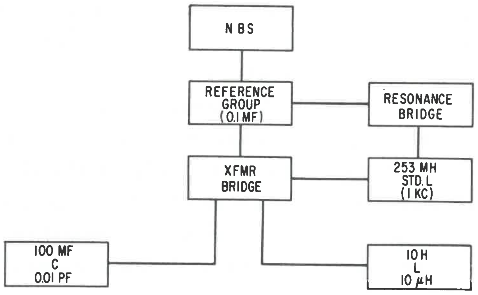

Fig. 6—Capacitance and inductance standardization based on one reference group of capacitors.

Using the transformer bridge, capacitance standards will be extended over the 11 decades from 0.01 pf to 100 μf. Then, using the transformer bridge as a resonance bridge, the transfer from capacitance to inductance will be made. A value of 253 mh is shown as a first step since this resonates with 0.1 μf at 1 kc. Then, again with the transformer bridge, inductance will be standardized over the range from 10 μh to 10 henries.

We are now evaluating the resonance bridge approach. Preliminary results, capable of considerable refinement, have checked the value of a 200-mh inductor to within 0.03 per cent of its value as established with a 100-mh standard certified by NBS to ±0.03 per cent.

Conclusions

The test results show that this bridge is capable of intercomparing inductances and capacitances at ratios as large as 10:1 to accuracies an order of magnitude better than certification limits of present standards. The total error accumulated over six or more decades of reactance is no more than a few thousandths per cent, exclusive of lead inductance and two-terminal capacitance effects.

Measurements of any odd value can be made over a spread of 100:1 using the same standard, and with all the accuracy discussed above. All need for reference standards at less than decade intervals is eliminated. When standards capable of certification to higher accuracy become available, the bridge will enable us to take advantage of this.

Adding the proposed resonance bridge technique should make it possible with one reference group of capacitors to standardize both capacitance and inductance at 1 kc over a range of six or more decades to an accuracy approaching 0.01 per cent. This is an improvement of 3 to 10 times over the present accuracies, plus a 17 to 1 reduction in direct reliance on NBS for certifications.

APPENDIX

Derivation of (1) and (4)

Writing the loop equations for Fig. 3(b) and solving for E_2:

$$E = j\omega[i_1(L_1 + L_2 + 2M) - i_2(L_2 + M)]$$

$$O = j\omega\left[-i_1(L_2 + M) + i_2\left(L_2 - \frac{1}{\omega^2 C_g}\right)\right].$$

Solving,

$$i_2 = \frac{E(L_2 + M)}{j\omega\left[(L_1L_2 - M^2) - \frac{1}{\omega^2 C_g}(L_1 + L_2 + 2M)\right]}$$

but $E_2 = i_2/j\omega C_g$, and substituting for i_2

$$E_2 = \frac{E(L_2 + M)}{(L_1 + L_2 + 2M) - \omega^2 C_g(L_1L_2 - M^2)}. \tag{1}$$

Substituting into (3) the relationships

$$\frac{L_1}{L_2} = \left[\frac{(N_1)}{(N_2)}\right]^2 \quad \text{and} \quad M = \sqrt{L_1L_2}$$

there is obtained

$$E_2 = \frac{E(L_2 + \sqrt{L_1L_2})}{L_1 + L_2 + 2\sqrt{L_1L_2}} = \frac{E\sqrt{L_2}(\sqrt{L_1} + \sqrt{L_2})}{(\sqrt{L_1} + \sqrt{L_2})^2}$$

$$\frac{E_2}{E} = \frac{\sqrt{L_2}}{\sqrt{L_1} + \sqrt{L_2}} = \frac{N_2}{N_1 + N_2} \tag{4}$$

Derivation of Errors Caused by Ground Capacitance, C_G, Associated with Series Balancing Resistance, R_D

Referring to Fig. 4(b), ignoring shunt capacitances, C_D and C_{sh}, the equivalent π impedance between 1 and 2 is

$$Z_{12} = \frac{-\frac{j}{\omega C_G}(R_S + j\omega L_S) - \frac{j}{\omega C_G}R_D + R_D(R_S + j\omega L_S)}{-j/\omega C_G}.$$

Reducing, and grouping reals and imaginaries:

$$Z_{12} = [R_S + R_D(1 - \omega^2 L_S C_G)] + j\omega(L_S + R_D R_S C_G)$$

with $C_G = 0$, $Z_{12}' = (R_S + R_D) + j\omega L_S$. Subtracting Z_{12}' from Z_{12} gives the error due to C_G.

$$\text{Error} = Z_{12} - Z_{12}' = (-\omega^2 R_D L_S C_G) + j\omega(R_D R_S C_G).$$

Derivation of (8) from Fig. 4(a)

$$\frac{1 - A}{A} = \frac{C_x + \omega^2 L_l C_x^2}{C_s + \omega^2 L_r' C_s^2}$$

where $L_r' = (L_r + L_D)$

$$C_x = \frac{(1 - A)}{(A)}C_s + \frac{(1 - A)}{A}\omega^2 L_r' C_s^2 - \omega^2 L_l C_x^2$$

$$= \frac{(1 - A)}{(A)}C_s + \Delta C_x$$

but

$$\frac{(1 - A)}{A} = \frac{C_x}{C_s}$$

to an accuracy sufficient for correction purposes. Then

$$\Delta C_x = \frac{C_x}{C_s}\omega^2 L_r' C_s^2 - \omega^2 L_l C_x^2$$

$$\Delta C_x = \omega^2 C_x(L_r' C_s - L_l C_x)$$

$$= \omega^2 C_x[C_s(L_r + L_D) - C_x L_l]. \tag{8}$$

Derivation of (9) from Fig. 5(a) and 5(b)

$$\text{Cos}\,\theta = \frac{(\Delta A)E}{|I|(\Delta R_x)/2} \cong \frac{R_x}{|Z_x|}$$

but

$$|I| = \frac{AE}{|Z_x|}.$$

Substituting for $|I|$ and solving for ΔA, the ratio error:

$$\Delta A = \frac{(\Delta R_x)AR_x}{2|Z_x^2|}. \tag{9}$$

Derivation of Shortcut Method of Readout Calculation

Assume a 10:1 comparison, where $L_x \cong 0.1\ L_s$

$$\frac{A}{1 - A} = \frac{L_x}{L_s} = \frac{0.1L_s + \Delta L_x}{L_s} = \frac{0.0909090\cdots + \delta}{0.9090909\cdots - \delta}$$

where ΔL_x is the departure of L_x from exactly 0.1 L_s, and δ is the departure of A from exactly 0.0909090 $\cdots$.

Solving for ΔL_x

$$\Delta L_x = \frac{1.1\delta L_s + \delta(\Delta L_x)}{0.9090909\cdots} = 1.21\delta L_s$$

since $\delta(\Delta L_x)$ is negligibly small. Substituting for ΔL_x in the first equation above, the true ratio becomes

$$\frac{A}{1 - A} = \frac{L_x}{L_s} = 0.1 + 1.21\delta.$$

Numerical example: let $L_S = 10$ h, $A = 0.0906818$ (0.25 per cent low)

$$\frac{A}{1 - A} = 0.1 + 1.21(0.0906818 - 0.09090909\cdots)$$

$$= 0.1 + 1.21(-0.0002273)$$

$$= 0.0997250 \text{ by slide rule.}$$

The exact value of the ratio is 0.09972505,